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O JEDNOM NAČINU TRETIRANJA JEDNADŽBE KONTINUITETA ILUSTRIRANIM SA GODIŠNJIM HODOM SALINITETA U SREDNJEM JADRANU

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INTRODUCTION

The application of the equation of continuity on turbulent current is not an easy one, because both measurements and computations give mean flow fields out of which the component with the shortest periods of fluctuations have been eliminated, so that their impact on the transport of specific value observed remains an open question. This difficulty is usually overcome using the coefficients of turbulence.

But, the coefficients of turbulence, although formaly clearly defined, are inconvenient for experimental elaboration and their physical meaning is, also, very limited and incomplete. This is do to the fact that their theory is based on those chareteristics of the flow field which are usually determined by the coefficients themselves (that is to say by the results obtained by their application) they having been introduced as a formal value determined exclusively through the characteristics of the flow field.

This led us to try to define physicaly more explicithy the component of the flow field which is treated by application of turbulence coefficients in the equations of continuity.

An attempt has been made here to determine the characteristic fluctuations in the flow field important for determination of field of the given specific values. The characteristic fluctuations in the flow field should be worked out observing the basic Fourier time series harmonic constituents in the field of the observed specific value.

General theoretical discussion in this paper has been illustrated by establishing a simple model of annual salinity fluctuations in the surface layer of the open central Adriatic. The quantitative results themselves are illustrative because we have not intended to give strictly quantitative observations.

THEORETICAL CONSIDERATIONS

The salinity field has been taken as a good example for the analyses brought out, the results being applicable to the field of any other specific conservative value.

Salinity field is uniquely determined in the given time-space domain by boundary conditions and initial conditions and by the equation of continuity with known flow field.

Let salinity, S (r, t) and current V (r, t), in the given domain of (r, t) arguments, satisfy Dirichlet conditions so that they might be expanded by any of the arguments in the convergent Fourier series.

Let the expansion by time be

$$S(r,t) = \sum_{k=-\infty}^{+\infty} S_k(r) \exp(ik\omega t),\ V(r,t) = \sum_{k=-\infty}^{+\infty} V_k(r) \exp(i\omega kt) \tag{1}$$

If $\partial s/\partial t$ and $\partial^2 s/\partial t^2$ also satisfy Dirichlet conditions then by their exspansion in Fourier series the convergent majorant M/k^2 is obtained for Fourier series S (r, t). This means that Fourier series S (r, t), according to Weierstrass criterion, is uniformly convergent, so that continuity equation

$$\partial S/\partial t + V \cdot \nabla S = 0 \tag{2}$$

can be written as

$$\sum_{k=-\infty}^{+\infty} i\,k\,\omega\,S_k \exp(ik\omega t) + \sum_{k=-\infty}^{+\infty} V_k \exp(ik\omega t) \cdot \sum_{k=-\infty}^{+\infty} \nabla S_k \exp(ik\omega t) = 0 \tag{3}$$

what is the same as

$$\sum_{n=-\infty}^{+\infty} (i\,n\,\omega\,S_n + \sum_{k=-\infty}^{+\infty} V_{n-k} \cdot \nabla S_k) \exp(in\omega t) = 0 \tag{4}$$

because the members $V_j \cdot \nabla S_k \exp[i(j+k)\omega t]$ appear as the products of series. If we take that $n = j + k$, then j together with k and n is uniquely determined. Since the series are infinite we might take (4) instead of (3).

Out of (4) follows

$$i\,n\,\omega\,S_n + \sum_{k-\infty}^{+\infty} V_{n-k} \cdot \nabla S_k = 0 \tag{5}$$

which is the sought equation that joins together the harmonic changes in flow and salinity fields.

Experimental data on the fluctuations of salinity in the sea may indicate the possibility of reducing the infinite sum of the equation (5) to the finite one.

The annual fluctuations of salinity, according to Neumann (Defant, 1961) for the Newfoudland Banks and the Azores in the North Atlantic, is presented in the figure 1.

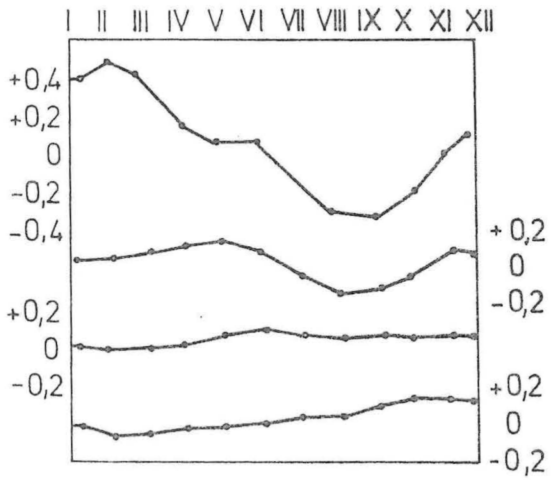

Fig. 1. — Annual salinity fluctuations in the Nort Atlantic in 1938. according to Neumann (anomalies in the localities between the Azores and Newfoundland Banks)

The monthly mean values for surface salinity at the Aldergrund light-ship anchor station in Baltic for the period 1926—1935, according to Neumann (Defant, 1961), are given in the table 1.

Table 1. The monthly mean values for surface salinity in Baltic

Month	I	II	III	IV	V	VI
Salinity	7.51	7.52	7.50	7.45	7.39	7.30
Month	VII	VIII	IX	X	XI	XII
Salinity	7.31	7.31	7.32	7.36	7.41	7.48

The values from the table 1. follow the function

$$S(t) = 7.41 + 0.103 \sin\left(\frac{2\Pi}{T} t + 66.8\right) + 0.006 \sin\left(\frac{2\Pi}{T} t + 15.4\right)$$

We can see from figure 1. and equation (6) that annual salinity fluctuations might be expressed, sufficiently approximate, by lower number of harmonics or that the infinite sum of equation (5) might be replaced by the finite one consisting of only few members.

If $\pm N$ in the last harmonic of $S(r, t)$ which is observed, then $\pm 2N$ is the last harmonic of flow field wihch has to be known in order to determine the field of $S(r, t)$ according to equation (5).

As it has been brought out in the introduction, the problems with the equation of continuity have usually been solved introducting the coefficients of turbulence and approximating the general flow field by the appropriate mean one. Here the problem has been reduced to solving the system of differential equations with space co-ordinates as indipendet variables. Thus we have obtained relatively simple definition of the properties of flow field which are to be observed through the nature of the fluctuations of the given specific value (here annual salinity fluctuations in the sea) and avoided the introduction of the coefficients of turbulence.

EXAMPLE WITH SURFACE LAYER OF THE CENTRAL ADRIATIC

According to everything that has been brought out, it is necessary first to examine the importance of each particular period of harmonic changes in the fluctuations of the value observed.

Table 2 shows first four harmonics of the annual salinity fluctuations calculated from the data taken monthly from the surface layer at the Stončica station (near Vis island) during 1970.

A-amplitude with cos in ‰	B-amplitude with sin in ‰	$\sqrt{A^2+B^2}$ amplitude	Arctang $\frac{A}{B}$
.33353	—.12970	.35786	—1.19992
—.18781	.11835	.22199	—1.00851
—.00155	—.19662	.19663	.00786
.09857	.08320	.12899	.86973

Annual salinity fluctuations might be expressed by the first harmonic only although with limited accuracy.

Let it be

$$S(r, t) = \sum_{n=-1}^{+1} S_n(r) \exp(i n \frac{2\Pi}{T} t) \quad (7)$$

From the average monthly circulation in the surface layer at Stončica (Zore-Armanda, 1966) flow field was determined as

$$V(r, t) = \sum_{n=-1}^{+1} V_n(r) \exp(i n \frac{2\Pi}{T} t) \quad (8)$$

and taken to be homogeneous in the observed area. Values are given in the Table 3. X-axis of the right Descartes co-ordinate system is taken to have the U_0 direction.

$U_0 = (2,0) \times 10$ km/year

Amplitude with cos $U_A = (-2, 3) \times 10$ km/year

Amplitude with sin $U_B = (3, -1) \times 10$ km/year

Water precipitated from the atmosphere affects little salinity fluctuations. Salinity fluctuations and precipitations for Stončica in 1967. are shown in the Figure 2. (sal.: Buljan and Zore-Armanda, 1967; precipitations: Publ. Savez. HMZ, 1967).

From the same data the coefficient of correlation between them has been calculated. It is + 0.4 what has no sense if only these tho values are taken into consideration (coefficient of correlation should have been negative because considerable increase in precipitations causes the salinity decrease).

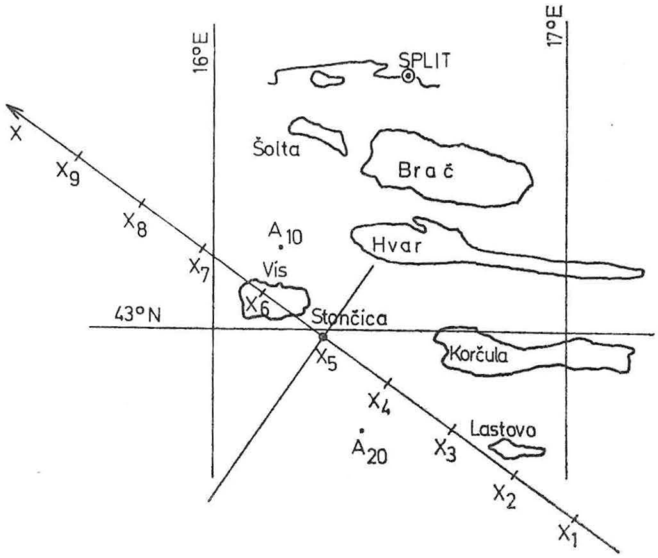

Fig. 2. — Surface salinity and precipitations at Stončica in 1967.

The pyknocline that occurs in summer beneath the surface layer, renders impossible any more intensive mixing with the surface, so that the influence of convection can be ignored.

Besides, because of the inflow of large masses of sea water of high salinity from the Mediterranean in the course of winter, the influence of convection is not the primary one.

According to the equation 5 and everything brought out, we can establish the system of equations of annual salinity fluctuations for Stončica with the meaning restricted by the discussion already carried out. It will be

$$-i\,\omega\, S_{-1} + U_0\, \partial S_{-1}/\partial x + U_{-1} \cdot \nabla S^0 = 0 \qquad (9)$$

$$U_{-1} \cdot \nabla S_1 + U_0\, \partial S_0/\partial x + U_1 \cdot \nabla S_{-1} = Q_0 \qquad (10)$$

$$i\,\omega\, S_1 + U_0\, \partial S_1/\partial x + U_1 \cdot \nabla S_0 = 0 \qquad (11)$$

where the allowed possibility of stationary convenction of salinity per unit time is equal to Q_0.

In the case when S_0 field is taken to be linear and known, and Q^0 an unknown function, the given system is going to be decomposed into three mutually independent ordinary differential equations.

Then the solution will be:

$$S(x, y, t) = [S_A(x = o, y) - \frac{U_B}{2\Pi} \cdot \nabla S_0(x, y)] \cos(\omega t - \frac{2\Pi}{U_0} x) +$$

$$+ [S_B(x = o, y) + \frac{U_A}{2\Pi} \cdot \nabla S_0(x, y)] \sin(\omega t - \frac{2\Pi}{U_0} x) +$$

$$+ \frac{V_B}{2\Pi} \cdot \nabla S_0(x, y) \cos \omega t - \frac{V_A}{2\Pi} \cdot \nabla S_0(x, y) \sin \omega t + S_0(x, y) \quad (12)$$

where the amplitudes with cosine and sine are marked by A and B indexes, and $S_0(x, y)$ is the given linear function of x and y, whereas $S_A(x = o, y)$ and $S_B(x = o, y)$ are any given functions of x and y.

In the course of solving the given system of differential equations, the potential dependence of U_0 on y would not change anything in the solution of (9)—(11) what is important for the observed area.

The co-ordinate system is given in the Fig. 3.

The constant gradient ∇S_0 has been determined from the data of the NAJADE Expedition in 1911. (NAJADE, 1911) for the stations A_{10}, A_{13} and A_{20}.

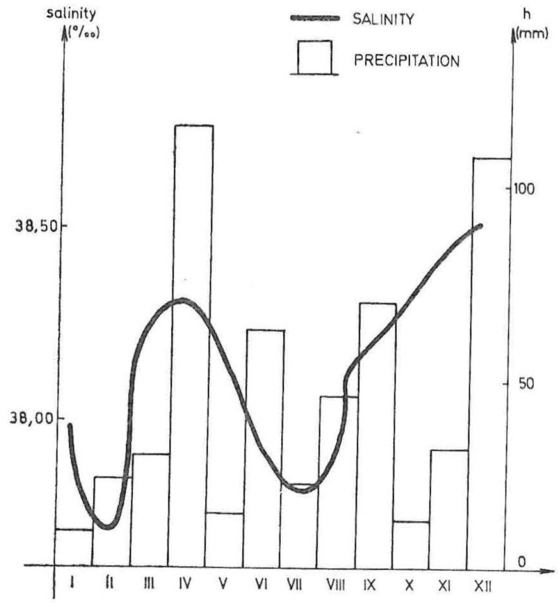

Fig. 3. — Co-ordinate system of the model

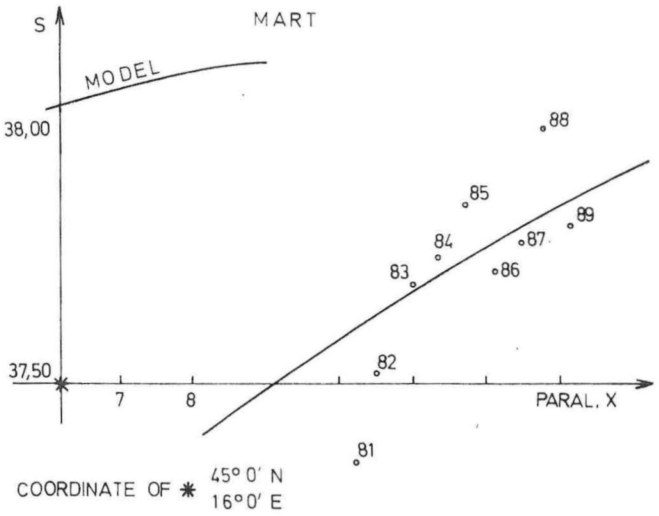

Fig. 4. — Salinity for y = 0 according to model

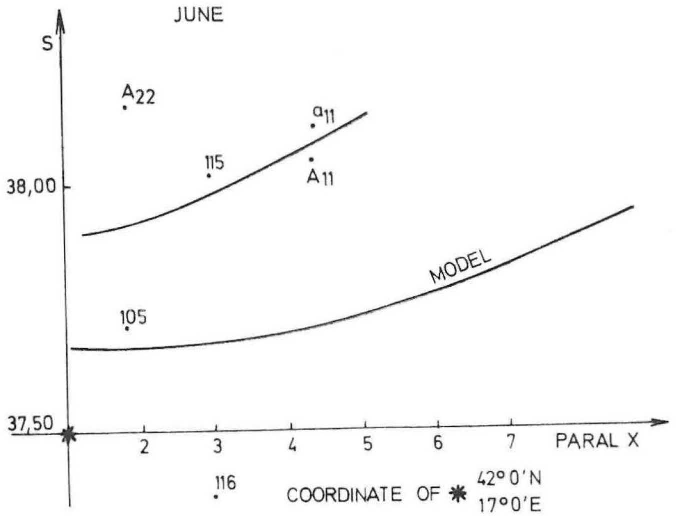

Fig. 5. — Salinity according to model and data for March

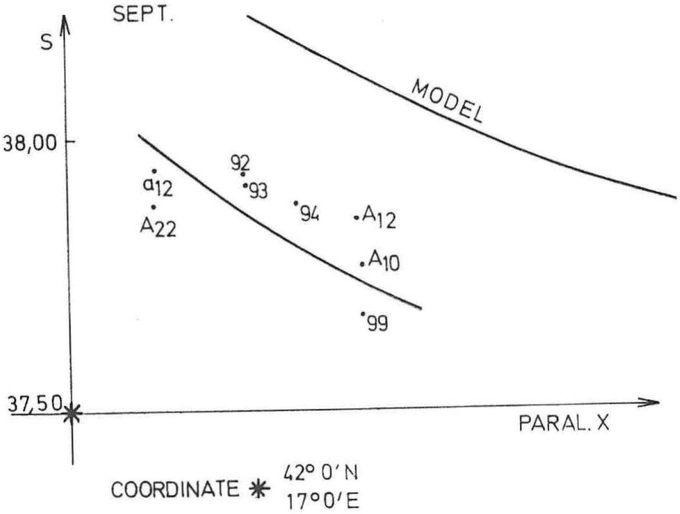

Fig. 6. — Salinity according to model and data for June

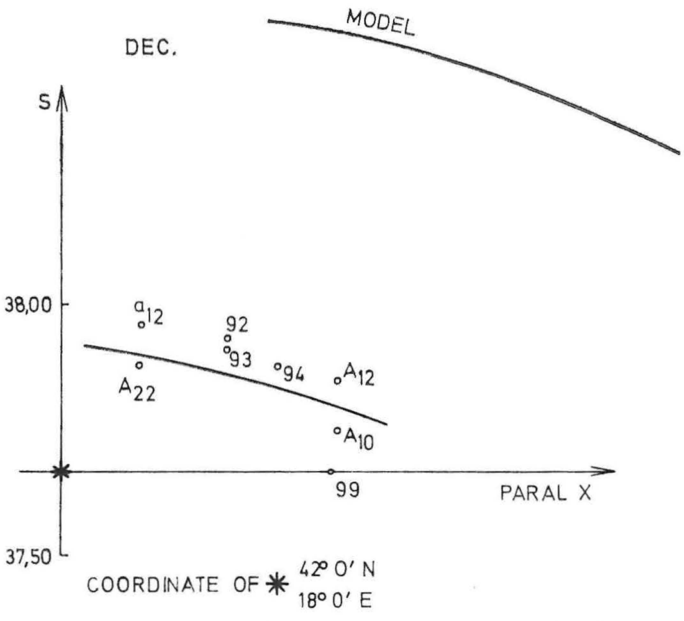

Fig. 7. — Salinity according to model and data for September

Results are given in the Table 4. The mean value of surface salinity at Stončica in 1967. has been taken for S_0 (x = o, y = o), in the same way as S_A (x = o, y = o) and S_B (x = o, y = o) have been determined in the Table 2.

$$\nabla S_0 = \left(\frac{S_0(A_{13}) - S_0(A_{20})}{x(A_{13}) - x(A_{20})}, \frac{S_0(A_{13}) - S_0(A_{10})}{y(A_{13}) - y(A_{10})}\right) = (-0.17,\ 0{,}57) \times 10^{-3} \quad ‰/\text{km}$$

$U_A \cdot \nabla S_0 = 2‰/\text{year} \qquad U_B \cdot \nabla S_0 = 1‰/\text{year}$

S (x., y = o, t_k) has been calculated according to the model where i = 1,9 and X_5 is at the zero point of the co-ordinate system which is at Stončica. $\Delta X = X_{i+1} - X_i = 17$ km and t_1 = 15. II, t_2 = 15. VI, t_3 = 15. IX, t_4 = 15. XII, have been taken. The rezults of the model are shown in the Figure 8.

According to the data collected by the NAJADE Expedition in 1911. for the stations chosen such as to lay allong the lines approximately parallel to x-axis of the model and as close as possible to it, the evaluation of salinity distribution has been given in the figures 5, 6, 7, and 4 and compared with the results obtained from the model. Good agreement between the trends of both distributions in all seasons is obvious.

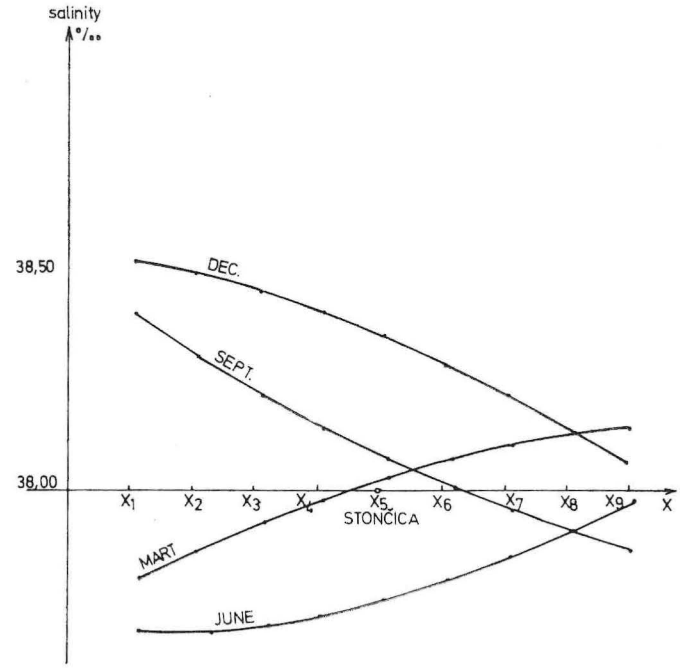

Fig. 8. — Salinity according to model and data for December

CONCLUSIONS

The process of approximative treatment of the problem of continuity, that has been brought out, is convenient when the fluctuations of the observed value might be presented by a lower number of harmonics in the domain of periods suitable for both experimental and theoretical treatment of flow field. In the case when two harmonics of the observed value are to be considered, ahe analytical treatment of the problem is a rather comprehensive task, so that when more than two harmonics are to be considered, it can be carried out only by the help of numerical method with computer.

It is likely that, by the process brought out, some characteristics of the turbulence coefficients could be theoretically examined.

It is necessary to point out the mathematical-analytical restrictions that had to be introduced. The most strict one was that second partial time derivation of the observed value fluctuations should exist and satisfy Dirichlet conditions.

ACKNOWLEDGEMENTS

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I wish, also, to thank dr. M. Zore-Armanda for her advice and help in reviewing and preparing the manuscript.

LIST OF SIMBOLS

i — imaginary unit
r — radius vector
x, y — horisontal coordinates
t — time
S — salinity
S_k — salinity harmonics
V — velocity vector
V_k — velocity vector harmonics
U_{1}, U_0, U_{1} — zero and firsts harmonics of V

SUMMARY

The process of approximative treatment of problem of continuity has been brought out. It is convenient when the fluctations of the observed value might be presented by a lower number of harmonics in the domain of periods suitable for both experimental and theoretical treatment of flow field.

It is likely that, by the process brought out, some characteristics of the turbulence coefficients could be theoretically examined.

O JEDNOM NAČINU TRETIRANJA JEDNADŽBE KONTINUITETA ILUSTRIRANIM SA GODIŠNJIM HODOM SALINITETA U SREDNJEM JADRANU

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KRATKI SADRŽAJ

U radu je razmatrana jedna mogućnost tretiranja jednadžbe kontinuiteta.

U prvom djelu rada definirane su one harmonijske komponente Fourier-ovog reda strujnog polja razvitog po vremenu koje su važne za određivanje polja zadate specifične veličine ako njen Fourierov red razvijen po povoljno odabranom osnovnom vremenskom intervalu brzo konvergira.

U drugom dijelu rada opći rezultati iz prvog dijela ilustrirani su postavljanjem jednostavnog modela godišnjeg hoda saliniteta u otvorenom srednjem Jadranu.

Iznijeti postupak aproksimativnog tretiranja problema kontinuiteta pogodan je onda kad se hod promatrane veličine može prikazati manjim brojem harmonika u području perioda pogodnih za eksperimentalno ili teorijsko tretiranje strujnog polja.

Analitičko tretiranje problema kada se moraju uzeti u obzir već i samo dva harmonika hoda promatrane veličine poprilično je opsežan posao, a kod većeg broja harmonika u obzir dolaze samo numeričke metode i rad s kompjuterom.

Iznijetim postupkom vjerovatno bi bilo moguće teorijski ispitati i neke karakteristike koeficijenata turbulencije.

Treba istaći matematičko-analitička ograničenja koja su morala biti uvedena. Svakako tu je najstrože ograničenje da druga derivacija hoda promatrane veličine mora postojati i zadovoljavati Dirichletove uvjete.

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