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INTRODUCTION

General aspects of the theory of hydrodinamic processes in the sea and atmosphere have been illustrated by a system of square partial differential hydrodynamic equations.

Mathematical methods of this system analysis have been developed as various approximative processes. Their choice has depended on the nature of the problem to be examined or on the way of problem's definition.

Usual way of development of approximative mathematical procedure of the sea and atmosphere hydrodynamic processes analysis is to linearize appropriately hydrodynamic equations, analyse linearized solutions and study their interaction in nonlinear solutions.

Up to now, the study of solutions of linearized hydrodynamic equations has been satisfactorily completed for the large scale of the sea and atmosphere hydrodynamic processes. Meanwhile, the study of their interaction is currently the subject of very intensive investigations.

This kind of plan of studies, however, leaves unanswered the theoretical question to what degree does the linearization, as a special condition, affect directly the image of processes physics (Eckart, 1960. p. 2).

Conformable to usual techniques of solving the linear systems of differential equations the question already asked may be formulated like this: if a part of time-space spectra of hydrodynamic values (flow field in the first place) has been determined in satisfactory approximation from the linearized hydrodynamic equations, taking the condition that all the hydrodynamic processes refer to the part of the spectrum observed, would the presence of hydrodynamic processes from other spectrum parts modify essentially the results obtained for this part of the spectrum.

As a part of this problem investigations this paper includes the study of advection member in acceleration, which is neglected in many of the studies of the sea and atmosphere hydrodynamics, whereas it is closely connected with the above mentioned question.

ON NEGLECTING OF ACCELERATION ADVECTIVE MEMBER

Vector flow field in the fluid *v (r, t)* is defined as momentum field of unit masses movement in the fluid with the gravity centre at *r* and time *t* (Hylleraas, 1970). In order to make the theoretical analyses or as their consequence, flow field is frequently represented either as finite or nonfinite series of uniformly convergent harmonic functions (or by multidimensional Fourrier series, or by the simple waves sum (Eckart, 1960). Assume that

$$v(r,t) = \sum_{k,\omega} V(k,\omega)\, e^{i(k \cdot r + \omega t)} \quad (1)$$

where $\omega = 2\Pi \frac{n_t}{T}$, $k = 2\Pi \left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right)$, $n_t, n_x, n_y, n_z = 0, \pm 1, \pm 2, \ldots$ and T is time interval during which the flow field is observed in the space domain $x \varepsilon [x_0, x_0 + L_x]$, $y \varepsilon [y_0, y_0 + L_x]$, $z \varepsilon [z_0, z_0 + L_z] \cdot V(k, \omega)$ is spatial-temporal spectrum of v (r, t) in the domain observed. If f (r, t) is the force that affects the unit mass of fluid with gravity centre at r and time, t, the equation will be

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = f$$

On the basis of (1) Equation (2) may be rewritten as

$$i \Omega V(k, \Omega) + \sum_{k,\omega} V_{(K-k,\, \Omega-\omega)} \cdot i k V_{(k,\omega)} = F(K, \Omega) \quad (3)$$

where the equation and its sum have been obtained as a form of convolution in the following manner:

$$v \cdot \nabla v = \sum_{k,\omega} V e^{(k,\omega)} \cdot \sum_{k',\omega'} i k' V_{k',\omega'} e^{i(k \times r + \omega t)} = \sum_{(k,\omega)} \sum_{(k',\omega')} V_{(k',\omega')} \cdot i k V_{i(k \cdot r + \omega' t)} e^{i[(k'+k) \cdot r + (\omega' + \omega) t]}$$

And by the introduction of $K = k' + k$, $\Omega = \omega' + \omega$ the sought form is obtained for K and Ω by which the equation has been dencomposed into an infinite number of equations like (3), taking account of linear independence of exponential functions. It can be seen in the Equation (3) that spectral component of the flow field depends on the whole of the flow field spectrum. Even in the case when the Equation (1) is not linearized by advective acceleration member neglecting, as done in some of the sea and atmosphere numerical models, hydrodynamic processes observed refer to an implicitly given part of the spectrum. That's why we have to unanswer the theoretical question of the nature of effect of hydrodynamic processes from the parts of the spectrum that are present in the nature but have not been taken into consideration in either numerical or analytical-theoretical analysis.

Assuming that hydrodynamic processes refer only to the part of the spectrum observed, neglecting advective acceleration term might be, as it ordinarily is, accounted for in many ways. One of them is, for example, that either the movement is slow or that the processes observed are of the longwave character (Hylleraas, 1970). Acceleration in Equation (2) may be written as follows:

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = \sum_{(k,\omega)} i\,\omega\, V_{k,\omega}\left[1 + \frac{k}{\omega} \cdot v(r,t)\right] e^{i(k \cdot r + \omega t)}$$

Assuming that hydrodynamic processes from the other parts of the spectrum are not present and as $\left|\frac{k}{\omega} \cdot v\right| \leqslant \left|\frac{k}{\omega}\right| |v|$ where $\left|\frac{k}{\omega}\right| = c$ and c is wave phase velocity, all the hydrodynamic processes with the phase velocity greatly exceeding the flow velocity in very good approximation, may be observed neglecting the advection in acceleration term. This kind of processes are sound waves, for example, whose theory builds on the above mentioned approximation. The longwave approximation is more comprehensive and so it has been chosen as a starting point of further considerations. We can assume that hydrodynamic processes from the longwave part of the spectrum are present and that flowing is slow enough that the change of position of the fluid particle observed in space per unit time is small in relation to the wave length and amplitude (which is small because of the less velocity). Thus the particle acceleration generated by the change of position in the space configuration of the flow field may be neglected. On the basis of the above mentioned assumptions the theoretical considerations of the process may be postulated so as to observe only the acceleration generated by the local fluctuations in the flow field.

The approximation of advection neglecting in acceleration term thus understood provides the basis for the first approximation in the sea and atmosphere hydrodynamics (Eckart, 1960). Further observations are focused on the importance of influences of the shortwave part of the spectrum on the longwave solutions obtained by the above mentioned approximation.

In the Equation (2) the force may be averaged by a spatial domain $V = l_x l_y l_z$ where $x \varepsilon [x_0, x_0 + l_x]$, $y \varepsilon [y_0, y_0 + l_y]$, $z \varepsilon [z_0, z_0 + l_z]$ is the domain by which the force is averaged so that we have

$$f_1 = \frac{1}{V} \int_V f\, dv \tag{5}$$

and

$$f = f_1 + f_2 \tag{6}$$

The spatial distribution spectrum f_1 will rapidly vanish for all the wave lengths lower than l_x, l_y and l_z, and it will remain very stightly changed for the higher wave lengths. Spectrum f_2 is complementary to the spectrum f_1 in relation to spectrum f.

It is already known that solutions that satisfy the equation

$$\frac{\partial v_1}{\partial t} = f_1 \tag{7}$$

are good approximate solutions for the flow field if $f_2 = 0$ and l_x, l_z, l_y, are high enough in relation to flow velocity and if in the initial conditions there is no component of flow field from the shortwave part of the spectrum. Solution v_1 will rapidly converge to zero in the shortwave part of the spectrum and also will the f_1 force.

If we write the v solution of Equation (2) in the form

$$v = v_1 + v_2 \tag{8}$$

the Equation (2) for $f_2 \neq 0$ will be

$$\frac{\partial v_2}{\partial t} + (v_1 + v_2) \cdot \nabla (v_1 + v_2) = f_2 \tag{9}$$

For the longwave part of the spectrum ($k \approx 0$) we will obtain

$$\left(\frac{\partial v_2}{\partial t} + v_2 \cdot \nabla v_2\right)_{k \approx o} = -(v_1 \cdot \nabla v_2)_{k \approx o} \tag{10}$$

if in the Equation (9) we accept the condition that v_2 is either of the order of magnitude of v_1 or lower, considering only those solutions of Equation (2) whose velocities v are small in comparison to l_x, l_y and l_z. The member $v_2 \cdot \Delta v_1$ may be neglected by the same order of approximations like done previously when only the hydrodynamic processes from the langwave part of the spectrum were present.

Like in the Equation (3)

$$\left(\frac{\partial v_2}{\partial t} + v_2 \cdot \nabla v_2\right)_{k \approx o} = \sum_{k \approx 0, \omega, K, \Omega} V_1 (k - K, \Omega - \omega) \cdot i K V_2 (K, \Omega) e^{i (k \cdot r + \Omega t)} \tag{11}$$

and as $k \approx 0$ is observed and V_1 differs from zero only for $k - K \sim 0$ so by the same order of approximations it may be written as follows

$$\left(\frac{\partial v_2}{\partial t} + v_2 \cdot \nabla v_2\right)_{k \approx o} = 0 \tag{12}$$

The Equation (12) indicates: *the approximate solutions for v obtained may differ from rigorous solution, in presence of disturbances from shortwave part of the spectrum within longwave approximation, for only one additive inertial flow field that may be analysed irrespective of the studied longwave part.*

Inertial field may be neglected by conveniently chosen boundary or initial conditions. It may also be observed as a stationary flow field homogeneous along its stream lines or in any other suitable way.

LIST OF SIMBOLS

i — imaginary unit
r — radius vector
x, y, z — space coordinates
t — time
V — velocity vector
v — velocity vector
V — velocity vector harmonics
k, k', K — wave number vector
ω, ω', Ω — circular frequencies
f, f_1, f_2 — force vector
F — force vector harmonics

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KRATKI SADRŽAJ

Advektivni član akceleracije kod dugih valova u strujnom polju može se zanemariti ako se promatraju samo ti hidrodinamički procesi. Da li je to moguće ako su prisutni i hidrodinamički procesi u kratkovalnom području strujnog polja, odnosno da li prisustvo tih procesa bitno modificira gore dobijena rješenja — pitanje je kojim se bavio ovaj rad.

Za komponentu strujnog polja iz kratkovalnog područja koja po svom intezitetu nije takova da bi advekcija njome u dugovalnom području bila reda veličine da se mora uvažiti, pokazano je da se dobijena rješenja u dugovalnom području mogu razlikovati prisustvom utjecaja iz kratkovalnog područja uz red veličine greške dugovalne aproksimacije samo za aditivno inercijalno strujno polje.

Rezultat je dobijen prikazom polja strujanja sumom prostih valova (multimenzionalnim Fourierovim redom) i primjenom oblika teorema konvolucije na advektivni član akceleracije da bi se analizirao red veličine utjecaja.

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