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Discussion of error resulting from sea-air interaction in results of harmonic analysis of the tides

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The paper analyses the selectivity of the basic formula of the method harmonic analysis of the tides considering the length of the sea-level time-series. It is demonstrated that the method for sufficiently long time-series eliminates the error resulting from sea-air interaction. In the discussion the sufficient condition is given to consider a stochastic signal resulting from sea-air interaction as a continuous stationary stochastic process in the sea-level time-series.

INTRODUCTION

The tides with high precision may be described with a vertically integrated REYNOLDS equations neglecting the nonlinear terms of advection of momentum and considering the sea as a homogeneous incompressible fluid. HANSEN (1966) and others with these equations obtained the numerical solutions which evidently agree with measurements. Considering the non-linear terms it can be demonstrated from numerical solutions that they have considerable effects only on the periods greater than few days (BONE, 1993). This can explain the dynamic theory of tides which teaches that the tide waves caused by tide-generating forces must have the period of these forces (DEFANT, 1961). For the analysis of current-meter data in order to obtain the tide-current ellipses using GONELLA (1972) method the same hypothesis may be used.

Let's consider the sea-level record. The method harmonic analysis of the tides is based on the formula

$$C(\omega_m) = \frac{1}{M\ T_m} \int_0^{M T_m} h(t)\, e^{-i\,\omega_m t}\, dt \tag{1}$$

where h(t) is the sea-level record in function of time t at given point, ω_m are the angular frequencies of the tide-generating forces and T_m are their periods, M is some sufficiently large natural number and $C(\omega_m)$ are estimations of the complex amplitudes of the tides.

In this paper formula (1) will be discussed considering h(t) as the sum of deterministic signal from tide-generating forces and some stochastic signal resulting from sea-air interaction. It is discussed the selectivity of method depending on the length of the time series.

HARMONIC ANALYSIS OF THE TIDES

Let's consider sea-level record defined as a sum of harmonic functions with the periods of the tide-generating forces and some stochastic signal caused from sea-air interaction

$$h(t) = \sum_{n=-N}^{N} C(\omega_n)\, e^{i\,\omega_n t} + X(t)\,,\quad t \in [0,T] \tag{2}$$

where X(t) is stochastic signal, N number of the considered set of the harmonics of the tide-generating forces and T is the length of the sea-level record. Defining (BONE, 1988)

$$Z_k(\omega_m) = \frac{1}{T_m} \int_{(k-1)\,T_m}^{k\,T_m} X(t)\, e^{-i\,\omega_m t}\, dt\,,\ k=1..M \tag{3}$$

and

$$S_M(\omega_m,\omega_n) = \frac{1}{M\,T_m} \int_{0}^{M\,T_m} e^{i\,(\omega_n-\omega_m)\,t}\, dt \tag{4}$$

where ω_m and ω_n are rotational frequencies of the tides generating forces and M is ordinal number T/T_m, the integral from equation (1) may be written in the form

second sum in equation (5), from the integral in equation (3) may be written

$$E\,Z_k(\omega_m) = \frac{1}{T_m} \int_{(k-1)\,T_m}^{k\,T_m} E\,X(t)\ e^{-i\,\omega_m t}\, dt \tag{7}$$

where E X(t) = 0 because h(t) is measured from mean sea level, i.e. $E\,Z_k(\omega_m) = 0$. For Z_k is naturally to assume ergodicity, i.e. that the second term in equation (5) converges to zero for M tending to infinity.

Lets $\rho(j) = E\ Z_k * Z_{k+j}$ and assuming $\rho(j > j_0) = 0$, i.e. that the oscillations of sea level under atmospheric influences after a sufficiently large time interval are stochastically independent, it is

$$\lim \frac{1}{J} \sum_{j=0}^{J} \rho(j) = 0$$

when $J \to \infty$ which is sufficient condition for required ergodicity

$$\lim \frac{1}{M} \sum_{k=1}^{M} Z_k = E\,Z_k = 0$$

when $M \to \infty$, i.e. for the sufficient large M the second sum in equation (5) is negligible (BONE, 1988).

$$\frac{1}{M\,T_m} \int_{0}^{M\,T_m} h(t)\, e^{-i\,\omega_m t}\, dt = \sum_{n=-N}^{N} C(\omega_n)\, S_M(\omega_m,\omega_n) + \frac{1}{M} \sum_{k=1}^{M} Z_k \tag{5}$$

In the discussion below are given the general condition that X(t) has to satisfy to be considered as a continuous stationary stochastic process. Lets X(t) be a continuous stationary stochastic process, i.e.

$$E\,X(t) = \text{const.}\,,\ E\,X(t)\,X(t+s) - [E\,X(t)]^2 = R(s) \tag{6}$$

where E denotes mathematical expectation and R(s) is the covariance function. Considering the

Physically it may be visualized as an atmosphere over the sea which is acting periodically with approximately constant intensity but shifting in the phase, i.e. $E\ |Z_k| \neq 0$ and $E\ Z_k = 0$.

In the classical approach to the method harmonic analysis of the tides this problem is not considered in the conditions that for the analysis of the sea-level the time-series should be sufficiently long (e.g. DEFANT, 1961).

Considering the first term in the equation (5) follows

$$S_M(\omega_m, \omega_n) = \frac{e^{i(\omega_n - \omega_m) M T_m} - 1}{i(\omega_n - \omega_m) M T_m} \tag{8}$$

Let's $\omega_m = \dfrac{2\,\Pi}{T_m}$, it is $S_M = 1$ for $T_m = T_n$ and $S_M \cong 0$ for $|T_m - T_n| M >> T_n$, i.e. for the sufficiently large M the base equation of the method harmonic analysis of tide (1) in good approximation defines $C(\omega_n)$ considering the first term in equation (2), which is well known (e.g. DEFANT, 1961). The time-series of length 369 is the most favorable interval for a great number of component tide with a short period (DEFANT, 1961). Fig. 1. gives the function $|S_M(T_m - T_n)|$.

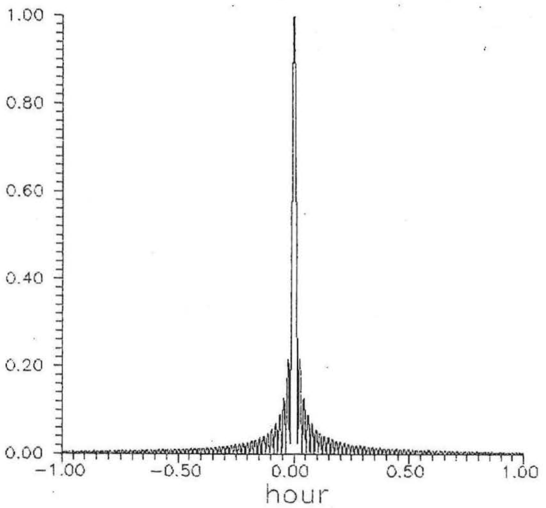

Fig. 1a. Resolution around M2 tidal constituent time-series 369 days

It is visible that the diurnal and semidiurnal components with difference in the periods greater than 0.01 hour are well separable. Some time are considered for estimations the time-series of length 29 day (DOODSON, 1954) where function S_M cannot separate the semidiurnal components with the difference in the period smaller than 0.2 hour and for the diurnal components smaller than 0.8 hour (as visible from Fig. 2.), but this is also sufficient for one basic analysis.

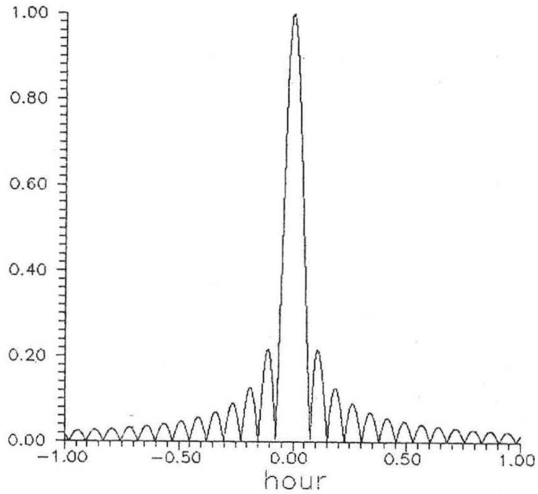

Fig. 1b. Resolution around O1 tidal constituent time-series 369 days

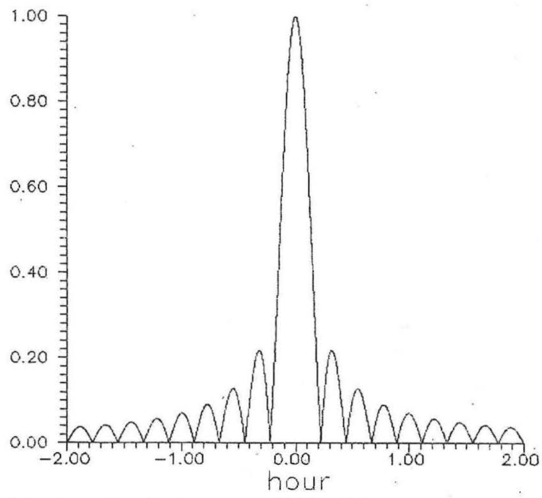

Fig. 2a. Resolution around M2 tidal constituent time-series 29 days

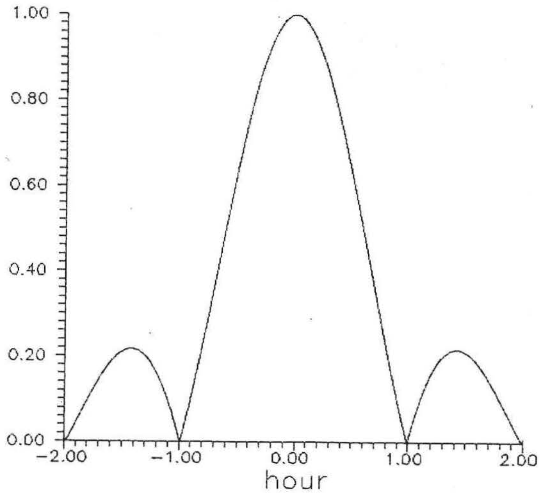

Fig. 2b. Resolution around O1 tidal constituent time-series 29 days

The time series of the length approximately 1 year seems to be sufficiently large in order to eliminate and here considered error from sea-air interaction, but this is very discutable if the time-series of the length 29 days is considered. Obviously, the long-period tides are largely influenced by meteorological perturbations of sea-level.

From equation (8) it appears that if the length of the time-series was chosen that it is approximately commensurable with the periods of the main tidal constituents (like in the cases of 29 and 369 day), $|S_M|$ will be small except for the considered one.

DISCUSSION

To analyse sea-level time-series there are three choices:

- to consider time-series as a solution of differential equation describing the physics of the changes in sea-level,
- to consider time-series as realization of a stochastic process, usually it is continuous stationary stochastic process,
- to consider time-series as a sum of term obtained solving differential equations and some term which is named stochastic fluctuations.

It is obvious that the last possibility is pragmatically actable and the first one should exclude some phenomena of physical interest.

Let's consider the function h(t) in equation (2). According to the CRAMER-LOVE theorem (e.g. WELSH, 1970) to any stationary stochastic process can be associated a spectral process with orthogonal increment, i.e.

$$h(t) = \int_{-\infty}^{\infty} e^{i\lambda t}\, dZ(\lambda) \tag{9}$$

where

$$E\, Z(\lambda)=0,\; E\left[(Z(\lambda_4)-Z(\lambda_3)\overline{(Z(\lambda_2)-Z(\lambda_1)))}\right]= 0 \tag{10}$$

superbar denoting complex-conjugate value and where $\lambda_1 \le \lambda_2 \le \lambda_3 \le \lambda_4$. According to BOCHNER theorem (e.g. WELSH, 1970), continuous function R(s) satisfying R(0)=1 is the covariance function of a continuous stationary stochastic if there exists a distribution function $F(\lambda)$ such that

$$R(s) = \int_{-\infty}^{\infty} e^{i s \lambda}\, dF(\lambda) \tag{11}$$

The condition that h(t) measured from mean sea-level is a continuous stationary process it is to assume that the integral

$$R(s) = \lim_{T\to\infty} \frac{\int_{-T/2}^{T/2} h(t)\, h(t+s)\, dt}{\int_{-T/2}^{T/2} h(t)^2\, dt} \tag{12}$$

exists, and that R(s) is continuous (BONE, 1988). According to LEVY inversion formula distribution function of the associated spectral process, $F(\lambda)$ may be written

$$F(\lambda_2)-F(\lambda_1) = \lim_{S\to\infty} \frac{1}{2\pi} \int_{-S/2}^{S/2} \frac{e^{-i\lambda_2 s} - e^{-i\lambda_1 s}}{-is}\, R(s)\, ds \tag{13}$$

with covariance function R(s) defined by equation (12).

One very general form for h(t) is (BONE, 1988)

$$h(t) = \sum_{j=-J}^{J} Z_j\, e^{i\omega_j t}\ ,\ t \in (-\infty,\infty) \tag{14}$$

with the spectral distribution function

$$F(\lambda)=\begin{cases} 0 & \text{for } \lambda \in (-\infty, \omega_{-J}) \\ \frac{1}{\sigma^2} \sum_{j=-J}^{P+1} |Z_j|^2 & \text{for } \lambda \in [\omega_P, \omega_{P+1}), P=-J..J-1 \\ 1 & \text{for } \lambda \in [\omega_J, \infty) \end{cases} \tag{15}$$

where $\sigma^2 = \sum_{-J}^{J} |Z_j|^2$ and where the integrals have sense according to the theory of generalized functions (distributions).

For the subset of the frequencies $\{\omega_j : j = -J..J\}$ with the periods of the tide-generating forces in general there are not finite time-series commensurable with the all these periods and h(t) can not be considered as periodic, i.e. statistically cannot be treated as circular processes (HANNAN, 1958) which make its statistical analysis very difficult. It is not so restrictive to assume X(t) as circular process (for example in the time-series long nearly 1 year) and this explains the importance of the method harmonic analysis of the tides with which it is possible to extract X(t) from h(t). It is obvious that the sufficient condition to consider X(t) as a continuous stationary process is that given above for h(t).

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Rasprava greške u metodi "harmonijska analiza morskih mijena" nastale interakcijom more-zrak

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KRATKI SADRŽAJ

U članku je analizirana selektivnost osnovne formule metode "harmonijska analiza morskih mijena" u funkciji dužine analiziranog vremenskog niza razine mora. Pokazano je da je ovom metodom za dovoljno dugi vremenski niz uklonjena greška koja nastaje interakcijom more-zrak. U raspravi je dat dovoljan uvjet da je u vremenskom nizu razine mora stohastički signal uzrokovan interakcijom more-zrak kontinuirani stacionarni proces.