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## ON THE PROFILE OF TIDAL CURRENTS IN THE NORTHERN ADRIATIC

O PROFILU STRUJA MORSKIH MIJENA U SJEVERNOM JADRANU

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Homogeneous vertical eddy viscosity coefficient is assumed for the tidal current in the northern Adriatic. The order of magnitude of this eddy viscosity coefficient is studied. The results were obtained comparing the measured current profiles with the profiles obtained from the assumed hydrodynamical model. The order of magnitude of the vertical eddy viscosity coefficient of  $100 \text{ cm}^2/\text{s}$  seems to be, in general, acceptable independently of season or position. The data do not demonstrate the existence of a separate bottom turbulent layer characteristic for low viscosity or suppression of negative rotary components characteristic for high viscosity.

### INTRODUCTION

The tidal current in the shallow sea in most of the cases can be in good approximation described by Ekman's equations (Nihoul et al., 1979).

The validity of these equations for the tidal current in the Northern Adriatic, where in the summer the strong stratification occurs, is discussable. However, it is assumed in this article. The major problem in these three-dimensional tidal computations is uncertainty about the vertical eddy viscosity coefficient (Tee, 1979). Here, the time independent and vertically homogeneous eddy viscosity is assumed. In this hydrodynamical model for the tidal current the hypothesis for eddy viscosity coefficient of different order of magnitude is tested on the current data collected from the open northern Adriatic in summer and in the coastal Vir sea area in winter.

Assuming the order of magnitude of the vertical eddy viscosity coefficient to be 10, 100 and  $1000 \text{ cm}^2/\text{s}$  the consistence of so obtained tidal current profiles from the model with data is discussed. In the case of the here considered time series it is also possible to examine the possible influence of

stratification and vicinity of the coast on the order of magnitude of the vertical eddy viscosity coefficient in the model. The result of this examination is an appropriate order of magnitude for the vertical eddy viscosity coefficient in the model. We have also been informed about possible tidal current profiles of this model.

## MATERIALS AND METHODS

The ordinary simplified equations of the tidal current hydrodynamics are Ekman's equations (Nihoul et al., 1979)

$$\begin{aligned} (1)a \quad & \partial_t(u) - f \cdot v = -g \cdot \partial_x(h) + \partial_z(N \cdot \partial_z(u)) \\ (1)b \quad & \partial_t(v) + f \cdot u = -g \cdot \partial_y(h) + \partial_z(N \cdot \partial_z(v)) \\ (1)c \quad & \partial_t(h) + \partial_x(\int_{-H}^0 u \cdot dz) + \partial_y(\int_{-H}^0 v \cdot dz) = 0 \end{aligned}$$

where  $\partial_t$ ,  $\partial_x$ ,  $\partial_y$  and  $\partial_z$  denote partial derivatives with respect to time  $t$ , horizontal space coordinates  $x$ ,  $y$  and vertical coordinate  $z$ ,  $g$  is acceleration of gravity,  $u$  and  $v$  denote horizontal velocity components,  $h$  is the free-surface elevation,  $H$  bottom depth,  $N$  the vertical eddy viscosity coefficient and  $f$  is Coriolis parameter. The numerical solutions obtained from these equations are represented by the time dependent velocity in the three-dimensional space and the sea surface elevation that depends on time and horizontal coordinates. The main problem is appropriate definition of vertical eddy viscosity coefficient (Tee, 1979). The vertical eddy viscosity coefficient can be assumed linear, i.e. time independent. The effect of the non-linearity in  $N$  on the tidal current profile was investigated by Johns (1976) in a numerical model of tidal channel. Following this result  $N$  is the function of position only in the three-dimensional space. It must be noted that knowledge of  $N$  dependence on the vertical coordinate is scarce because sufficiently adequate measurements and developed theory do not exist (Tee, 1979).

In this the possibility and significance of the application of the time independent and vertically homogeneous eddy viscosity coefficient to the tidal current in the open northern Adriatic and in the coastal Adriatic discussed.

From the Ekman's equations (1) Tee (1979) obtained the following solution for the tidal current profile of the vertically homogeneous and time independent vertical eddy viscosity coefficient

$$\begin{aligned} (2)a \quad & u\omega = (D \cdot A/a) \cdot \cosh(a \cdot \zeta) + (D \cdot B/b) \cdot \cosh(b \cdot \zeta) + C_1 \\ (2)b \quad & v\omega = -i \cdot \{(D \cdot A/a) \cdot \cosh(a \cdot \zeta) - (D \cdot B/b) \cdot \cosh(b \cdot \zeta)\} + C_2 \end{aligned}$$

where  $\zeta = z/D$ ,  $D$  denotes bottom depth and where the velocity  $(u, v)$  is expanded in Fourier series  $(u, v) = \sum_{\omega} (u\omega, v\omega) \cdot \exp(i \cdot \omega \cdot t)$  ( $\sum_{\omega}$  denotes summation over circular frequencies  $\omega$ ) and where there are

$$\begin{aligned} a &= (1 + i) \cdot D \sqrt{(\omega + f)/(2 \cdot N)}, & b &= (1 + i) \cdot D \sqrt{(\omega - f)/(2 \cdot N)} \\ C_1 &= -\{(D \cdot A/a) \cdot \cosh(a) + (D \cdot B/b) \cdot \cosh(b)\} \\ C_2 &= i \cdot \{(D \cdot A/a) \cdot \cosh(a) - (D \cdot B/b) \cdot \cosh(b)\} \end{aligned}$$

For  $(U\omega, V\omega) = \int_{-1}^0 (u\omega, v\omega) \cdot d\zeta$ , there are

$$D \cdot A/a = -(U\omega + i \cdot V\omega)/(2 \cdot L_1), \quad L_1 = \cosh(a) - \sinh(a)/a$$

$$D \cdot B/b = -(U\omega - i \cdot V\omega)/(2 \cdot L_2), \quad L_2 = \cosh(b) - \sinh(b)/b$$

In the Tee's (1979) article  $(u, v)$  is defined as

$(u, v) = \text{Re} \{(u\omega, v\omega) \cdot \exp(i \cdot \omega \cdot t)\}$ . The definition which is used here differs from Tee's definition. The Tee's definition is more restrictive and does not give the possibility to introduce Gonella's rotary components which are the basis of further discussion in this article. As the formalism used by Tee in order to obtain the equations (2) is the same and for here considered case we refer to Tee's article for the proof of this equations.

Using a rotary component method (Gonella, 1972) there are

$$s\omega p = u\omega + i \cdot v\omega \quad \text{cnjg}(s\omega m) = \text{cnjg}\{u(-\omega) + i \cdot v(-\omega) = u\omega - i \cdot v\omega$$

( $s\omega p$  positive and  $s\omega m$  negative rotary component of angular frequency  $\omega$ ,  $\text{cnjg}$  denotes complex conjugate) and  $S\omega p = U\omega + i \cdot V\omega$ ,  $\text{cnjg}(S\omega m) = U\omega - i \cdot V\omega$ . In this case it may be written

$$2 \cdot D \cdot A/a = -S\omega p/L_1 \quad \text{and} \quad 2 \cdot D \cdot B/b = -\text{cnjg}(S\omega m)/L_2$$

wherefrom

$$(3)a \quad |s\omega p|/|S\omega p| = f\omega p(\zeta, D, \omega, N) = |\{\cosh(a \cdot \zeta) - \cosh(a)\}/L_1|$$

$$(3)b \quad |s\omega m|/|S\omega m| = f\omega m(\zeta, D, \omega, N) = |\{\cosh(b \cdot \zeta) - \cosh(b)\}/L_2|$$

The rotation of barotropic current hodograph does not give any information on current hodograph at the actual depth. This is due to the fact that vertical scales of positive and negative rotary components are not the same. Functions  $f\omega p$  and  $f\omega m$  describe how this rotary components are distributed vertically for an arbitrary barotropic current.

The characteristic bottom depth  $D$  is 50 m for the Northern Adriatic and its coastal area. The tidal current profile is discussed so that  $\omega$  denotes diurnal and semidiurnal rotary frequencies. It is suitable to obtain the functions  $f\omega p$  and  $f\omega m$  for  $N = 10, 100, 1000 \text{ cm}^2/\text{s}$ . They differ qualitatively. The graphs are given in figs. 1-6). The Ekman layer thickness is 4 m for  $N = 10 \text{ cm}^2/\text{s}$  and 14 m and 45 for  $N = 100 \text{ cm}^2/\text{s}$  and  $N = 1000 \text{ cm}^2/\text{s}$  respectively. In the case  $N = 10 \text{ cm}^2/\text{s}$  the motion can be classified as ideal fluid motion with turbulent bottom layer. For  $N = 100 \text{ cm}^2/\text{s}$  and  $N = 1000 \text{ cm}^2/\text{s}$  the influence of turbulent bottom layer is presented in the entire water column from bottom to surface. In the case  $N = 1000 \text{ cm}^2/\text{s}$  the negative rotary components are suppressed and only positive rotary component is identifiable.

The current meter data from stations Panon and Trata (see fig. 7) were studied in order to discuss their compatibility with the model profiles. The former are the data for the open northern Adriatic in summer and the latter are the data from the coastal Adriatic in winter. By the method of harmonic analysis of the tide (e.s. Defant, 1961) and rotary component method (Gonella, 1972) the values  $s\omega p$  and  $s\omega m$  are obtained.

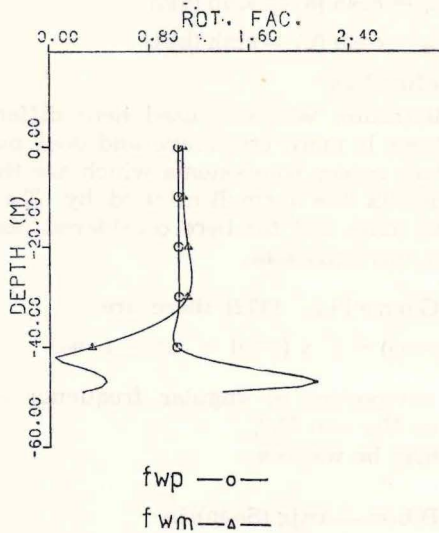


Fig. 1.  $f_{wp}$  and  $f_{wm}$  for semidiurnal tidal components,  $D = 50$  m,  $N = 10$  cm<sup>2</sup>/s.

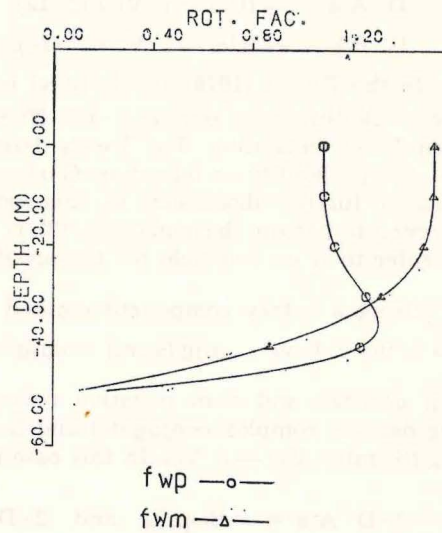


Fig. 2.  $f_{wp}$  and  $f_{wm}$  for semidiurnal tidal components,  $D = 50$  m,  $N = 100$  cm<sup>2</sup>/s.

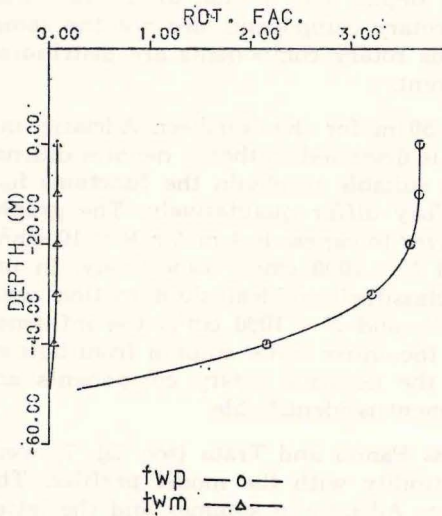


Fig. 3.  $f_{wp}$  and  $f_{wm}$  for semidiurnal tidal components,  $D = 50$  m,  $N = 1000$  cm<sup>2</sup>/s.

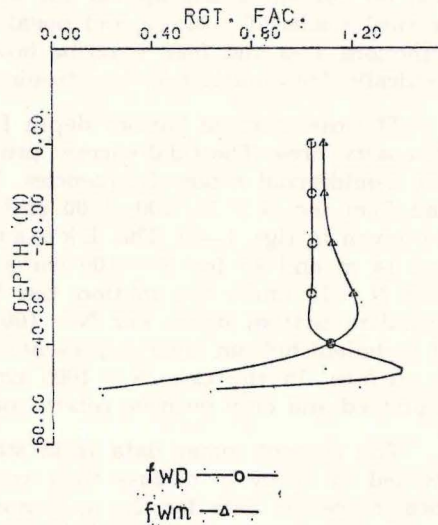


Fig. 4.  $f_{wp}$  and  $f_{wm}$  for diurnal tidal components.  $D = 50$  m,  $N = 10$  cm<sup>2</sup>/s.

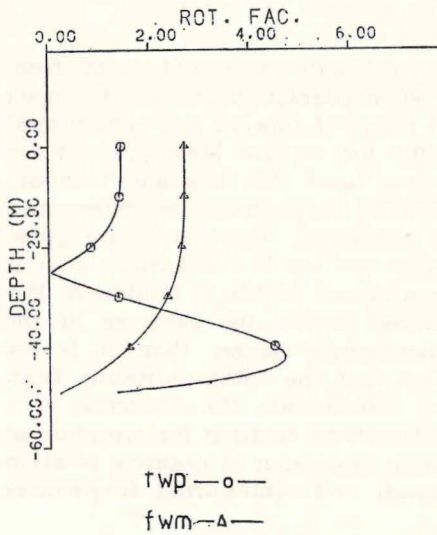


Fig. 5.  $f_{wp}$  and  $f_{wm}$  for diurnal tidal components.  $D = 50$  m,  $N = 100$   $\text{cm}^2/\text{s}$ .

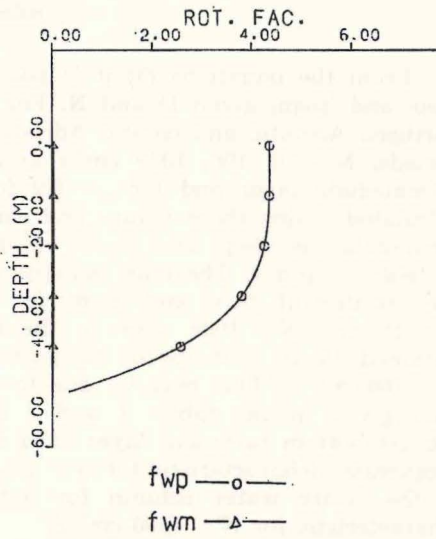


Fig. 6.  $f_{wp}$  and  $f_{wm}$  for diurnal tidal components.  $D = 50$  m,  $N = 1000$   $\text{cm}^2/\text{s}$ .

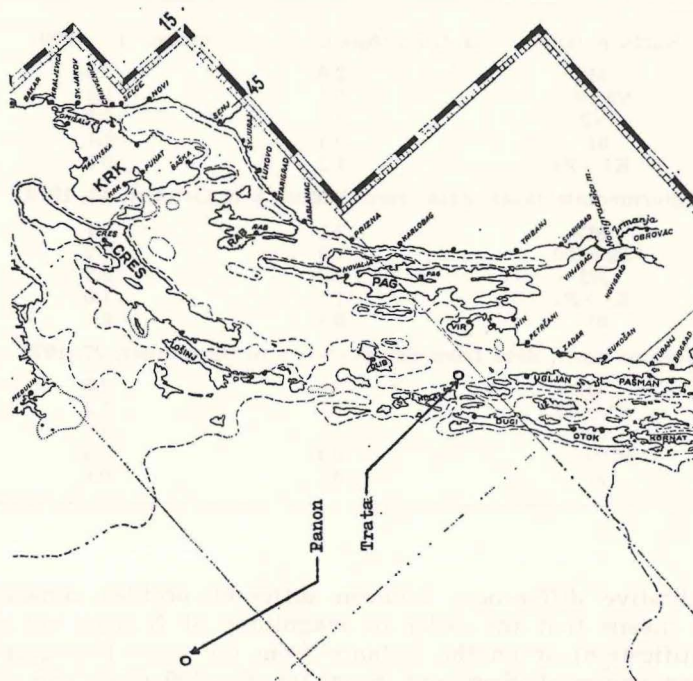


Fig. 7. Positions of the current-meter stations

## RESULTS

From the equations (3) it is possible to calculate  $|S_{\omega p}|$  and  $|S_{\omega m}|$  from  $|s_{\omega p}|$  and  $|s_{\omega m}|$  given  $D$  and  $N$ . For  $D = 50$  m characteristic for the open northern Adriatic and coastal Adriatic and for  $\omega$  of diurnal and semidiurnal periods,  $N = 10, 100, 1000$   $\text{cm}^2/\text{s}$  and  $\zeta = 0.1$  for surface layer,  $\zeta = 0.5$  for intermediate layer and for  $\zeta = 0.9$  for bottom layer, the  $f_{\omega p}$  and  $f_{\omega m}$  are calculated. Using these values and corresponding  $|s_{\omega p}|$  and  $|s_{\omega m}|$  determined from data the  $|S_{\omega p}|$  and  $|S_{\omega m}|$  values are calculated. The results are given in tables 3 and 4. The most consistent results are for  $N = 100$   $\text{cm}^2/\text{s}$ , that is the variance of  $|S_{\omega p}|$  and  $|S_{\omega m}|$  for the considered profile is minimum. For  $N = 10$  and  $N = 1000$   $\text{cm}^2/\text{s}$  in the considered profile the variance of the obtained  $|S_{\omega p}|$  and  $|S_{\omega m}|$  have been significantly larger than it is for  $N = 100$   $\text{cm}^2/\text{s}$ . This may be due to the fact that the obtained results from data given in the tables 1 and 2 do not demonstrate the existence of a distinct bottom turbulent layer with marked positive rotation for semidiurnal frequencies (characteristic for  $N = 10$   $\text{cm}^2/\text{s}$ ) or suppression of negative rotation in the entire water column for both diurnal and semidiurnal frequencies (characteristic for  $N = 1000$   $\text{cm}^2/\text{s}$ ).

Table 1. Rotary-components of the tidal current for the station Panon

Tidal component	$ s_{\omega p} $ cm/s	$ s_{\omega m} $ cm/s
Surface layer, data from August, 8 to October 17, 1979		
M2	2.9	2.4
S2+K2	1.3	1.2
N2	0.5	0.3
O1	0.4	0.1
K1+P1	1.2	0.8
Intermediate layer, data from August, 8 to October, 17, 1979.		
M2	2.1	2.1
S2+K2	1.2	1.2
N2	0.2	0.2
K+P1	1.1	1.0
O1	0.5	0.3
Bottom layer, data from September, 7 to September, 27, 1979.		
M2	2.2	2.5
S2+K2	1.1	1.2
N2	2.1	2.0
K1+P1	0.4	0.8
O1	0.7	0.6

No qualitative differences between different profiles considered were found. This means that the order of magnitude of  $N$  does not depend on season (stratification) or on the distance from the coast (topography).

The mean values of  $|S_{\omega p}|$  and  $|S_{\omega m}|$  for  $N = 100$   $\text{cm}^2/\text{s}$  and semidiurnal frequencies demonstrate characteristic positive rotation in the open northern

Table 2. Rotary-components of the tidal current for the station Trata

Tidal component	$ s\omega p $ cm/s	$ s\omega m $ cm/s
Surface layer, data from February, 22 to March, 27, 1979.		
M2	4.8	1.6
S2+K2	2.2	1.1
N2	0.7	0.4
K1+P1	1.5	0.7
01	0.6	0.5
Bottom layer, data from February, 28 to March, 27, 1979.		
M2	2.5	2.1
S2+K2	1.0	1.1
N2	1.4	0.4
K1+P1	1.4	0.9
01	0.5	0.5

Table 3a. Depth-averaged positive rotating rotary tidal current components  $|S\omega p| = |s\omega p|/f\omega p$  for the station Panon

tidal comp.	layer	N = 10 cm <sup>2</sup> /s $ S\omega p $ cm/s	N = 100 cm <sup>2</sup> /s $ S\omega p $ cm/s	N = 1000 cm <sup>2</sup> /s $ S\omega p $ cm/s
M2	surface	2.8	2.7	0.7
	intermediate	2.0	1.8	0.6
	bottom	0.5	2.5	1.6
S2+K2	surface	1.3	1.2	0.3
	intermediate	1.2	1.0	0.3
	bottom	0.3	1.2	0.8
N2	surface	0.5	0.5	0.1
	intermediate	0.2	0.2	0.1
	bottom	0.5	2.4	1.6
K1+P1	surface	1.2	1.8	0.3
	intermediate	1.1	6.1	0.1
	bottom	0.3	0.1	0.3
01	surface	0.4	0.3	0.1
	intermediate	0.5	0.2	0.5
	bottom	0.5	0.2	0.5

Adriatic as in the Taylor's solution for the reflection of Kelvin wave of semidiurnal frequencies in a semiclosed bay which qualitatively describe the semidiurnal tides in the northern Adriatic.

Table 3b. Depth-averaged negative rotating rotary tidal current components  
 $|S\omega m| = |s\omega m|/f\omega m$  for the station Panon

tidal comp.	layer	N = 10 cm <sup>2</sup> /s $ S\omega p $ cm/s	N = 100 cm <sup>2</sup> /s $ S\omega p $ cm/s	N = 1000 cm <sup>2</sup> /s $ S\omega p $ cm/s
M2	surface	2.2	1.5	40.0
	intermediate	1.9	1.4	35.0
	bottom	8.6	2.8	125.0
S2+K2	surface	1.1	0.7	20.0
	intermediate	1.1	0.7	20.0
	bottom	4.1	1.3	60.0
N2	surface	0.3	0.2	5.0
	intermediate	0.2	0.1	3.0
	bottom	6.9	2.2	100.0
K1+P1	surface	0.7	0.3	26.0
	intermediate	0.1	0.4	33.0
	bottom	1.1	0.9	80.0
01	surface	0.1	0.0	36.0
	intermediate	0.2	0.1	38.0
	bottom	0.8	0.7	60.0

Table 4a. Depth-averaged positive rotating rotary tidal current components  $|S\omega p| = |s\omega p|/f\omega p$  for the station Trata

tidal comp.	layer	N = 10 cm <sup>2</sup> /s $ S\omega p $ cm/s	N = 100 cm <sup>2</sup> /s $ S\omega p $ cm/s	N = 1000 cm <sup>2</sup> /s $ S\omega p $ cm/s
M2	surface	4.7	4.4	1.2
	bottom	0.6	2.8	1.8
S2+K2	surface	2.1	2.0	0.6
	bottom	0.2	1.1	0.7
N2	surface	0.7	0.6	0.2
	bottom	0.3	1.6	1.0
K1+P1	surface	1.5	1.0	0.3
	bottom	1.0	0.3	0.9
01	surface	0.6	0.4	0.1
	bottom	3.6	0.1	0.3



Table 4b. Depth-averaged negative rotating rotary tidal current components  
 $|S_{\omega m}| = |s_{\omega m}|/f\omega m$  for the station Trata

tidal comp.	layer	N = 10 cm <sup>2</sup> /s  S <sub>ωp</sub>   cm/s	N = 100 cm <sup>2</sup> /s  S <sub>ωp</sub>   cm/s	N = 1000 cm <sup>2</sup> /s  S <sub>ωp</sub>   cm/s
M2	surface	1.5	1.0	27.0
	bottom	7.2	2.3	105.0
S2+K2	surface	1.0	0.8	18.0
	bottom	3.8	1.2	55.0
N2	surface	0.4	0.2	38.0
	bottom	1.4	0.5	20.0
K1+P1	surface	0.6	0.3	23.0
	bottom	1.2	1.0	90.0
01	surface	0.5	0.2	17.0
	bottom	0.7	0.5	50.0

### CONCLUSIONS

Accepting the time independent and vertically homogenous vertical eddy viscosity coefficient in the Ekman's equations the obtained results for the profile of tidal currents in the open northern Adriatic and in the Adriatic coastal areas demonstrate neither separate bottom turbulent layer, characteristic for the low viscosity of vertical eddy viscosity coefficient  $N = 10 \text{ cm}^2/\text{s}$ , nor suppression of negative rotary components characteristic for high viscosity of  $N = 1000 \text{ cm}^2/\text{s}$ . This gives the order of magnitude  $N = 100 \text{ cm}^2/\text{s}$ . The viscosity is not in the order of magnitude dependent season (stratification) nor on the vicinity of the coast (topography).

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## O PROFILU STRUJA MORSKIH MIJENA U SJEVERNOM JADRANU

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## KRATKI SADRŽAJ

Pretpostavljen je stacionaran i vertikalno homogen koeficijent vertikalne turbulentne viskoznosti pa je studiran red veličine tog koeficijenta za struje morskih mijena u otvorenom i priobalnom sjevernom Jadranu. Rezultati su dobijeni usporedbom teorijskih profila računatih za različite redove veličina koeficijenta viskoznosti sa profilima dobijenim kao rezultat mjerenja. Za red veličine koeficijenta viskoznosti dobivena je vrijednost  $N = 100 \text{ cm}^2/\text{s}$  koja je nezavisna o položaju ili sezoni. Podaci nisu pokazali postojanje izdvojenog turbulentnog sloja karakterističnog za slabu viskoznost  $N = 10 \text{ cm}^2/\text{s}$  kao ni gušenje pozitivno rotirajuće rotacione komponente karakteristične za jaku viskoznost  $N = 1000 \text{ cm}^2/\text{s}$ .